COMPOSITIONAL APPLICATION OF A CHAOTIC DYNAMICAL SYSTEM FOR THE SYNTHESIS OF SOUNDS

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ABSTRACT

The paper presents a review of compositional application developed in the last years using a chaotic dynamical system in different sound synthesis processes. The use of chaotic dynamical systems in computer music has been a widespread practice for some time now. The experimentation presented in this work shows the use of a specific chaotic system: the Chua's oscillator, within different sound synthesis methods. A family of new musical instruments has been developed exploiting the potential offered by the use of this chaotic system to produce complex timbres and sounds. The instruments have been used for the creation of musical pieces and for the realization of live electronics performances.

1. INTRODUCTION

Mathematics and Music are strictly tied since the early foundation of Western culture. Since 80s, chaos and fractal geometry have strongly affected the development of a new field of musical research, with the use of non-linear dynamical systems. Many musical researchers tried using these systems as melodic pattern generators[1] and for audio synthesis. In this case, they can be used either as sound material or to create control structure. The chaotic behaviors exhibited by such systems are of potential interest to composers who work whit computers, because they offer a mean of endowing computer-generated music with certain natural qualities not attainable by other means[2]. Furthermore, non-linear functions and Music exhibit a comparable degree of self-similarity or autocorrelation inspiring to use these of systems in the practice of computer music for creative purposes [3].

Several non-linear dynamical systems have been used in the literature for the generation of sounds and music. We can generate deterministic chaotic signals either by finding numerical solutions to differential equations or by using first return maps. Among all these systems, Chua's oscillator is a non-linear circuit exhibiting a chaotic behavior, which provides a great family of strange attractors [4, 5, 6]. Because of the wide range of behaviors exhibited by this chaotic system many researchers and musicians have used it in musical applications, both for melodic pattern generation and for sound synthesis.

In the applications of Chua's oscillator to sound synthesis two approaches are possible [7]: apply the chaotic system as one component of a physical instrument's simulation (Chua's oscillator is mainly used as non-linear excitation source coupled with a linear resonator) or apply the chaotic system as a signal generator exploring the potential musical properties of the circuit. In the remainder of this work we will present some applications that explore the possibilities of using both of these approaches.

In the '90, Rodet used Chua's circuit and the time-delayed Chua's circuit, a slightly modified circuit, in sound synthesis processes based on physical modeling techniques [8, 9]. In this kind of application, chaotic signals are used because they cover a large space of sonic behaviors: from sounds perceived as harmonic without noise up to an essentially noisy sounds. This is a very important feature for sound synthesis since the creation of noise and irregularities of traditional instruments has always been difficult and unsatisfactory by using traditional computer music methods [10]. The wide range of behaviors provided by chaotic dynamical systems allows to obtain signals exhibiting many different properties, such as a clearly perceived pitch, an intermittent behavior between periodic and noisy sounds, complex combinations of harmonic, inharmonic and noise components. The simultaneous presence of sinusoidal components and noise in the signal is very interesting since this occurs for the majority of acoustic instruments and because this is relatively difficult to model in a way which is useful for musical purposes. Complex sounds produced by a chaotic dynamical system can be perceived in many different ways, for example, as a pitched sound with a tremolo or vibrato effect or with a complex time evolution of timbre. Furthermore, Chua's circuit has been used also as sound generator for both pitched and noisy sounds. Mayer-Kress et al. [11] tried to explore some classes of Chua's circuit attractors with respect to their auditory display and musical properties. They point out how the Chua's circuit provides filter-likes effects that occurs when a change in one of parameters of the circuit does not affect the pitch of the resulting waveform but redistributes the spectral energy, attenuating some frequency bands while amplifying others.

In the remainder of the paper, a series of application of the Chua's oscillator within different sound synthesis methods will be presented. The visual programming environment Pure Data (Pd) was used to develop and create a set of patches providing a family of new musical instruments that exploit the potential offered by the use of this chaotic system to produce complex sounds. The paper is organized as follows: Section 2 presents Chua's oscillator and its numerical simulation made using Pure Data. Section 3 presents some reflections about how to transform in sounds the time series produced by Chua's oscillator. Section 4 presents the compositional applications: a prototype, made in Pure Data, of a synthesizer based on the use of the Chua's oscillator in a process of synthesis by frequency modulation and the sound synthesis processes applied to create a performance for live electronics exploring the possibilities of using the sounds produced by a chaotic system.

2. CHUA'S OSCILLATOR

Chua's oscillator [4, 5, 6] is a canonical system for research in chaos, since it can be realized in a real-world setting as a simple electronic circuit. The system has three degrees of freedom, and is described by three state equations. However, we use a dimensionless model to simulate it, with six parameters: $\alpha, \beta, \gamma, a, b, k$ corresponding to the ratios between two or more physical components. Equation (1) shows the set of three differential equations defining the dimensionless model of the Chua's oscillator.

$$\begin{cases} \frac{dx}{d\tau} = k\alpha(y - x - f(x)) \\ \frac{dy}{d\tau} = k(x - y + z) \\ \frac{dz}{d\tau} = -k(\beta y + \gamma z) \end{cases} \quad (1)$$

where $f(x)$ in equation (1) is defined as follow:

$$f(x) = bx + \frac{1}{2}(a - b)\{|x + 1| - |x - 1|\} \quad (2)$$

Solving this system of differential equations allows us to simulate the evolution of the Chua's oscillator in three-dimensional phase space, made up by the variables: x, y, z. The qualitative behavior of the oscillation is controlled by the six input parameters $\alpha, \beta, \gamma, a, b, k$ with $k = \pm 1$.

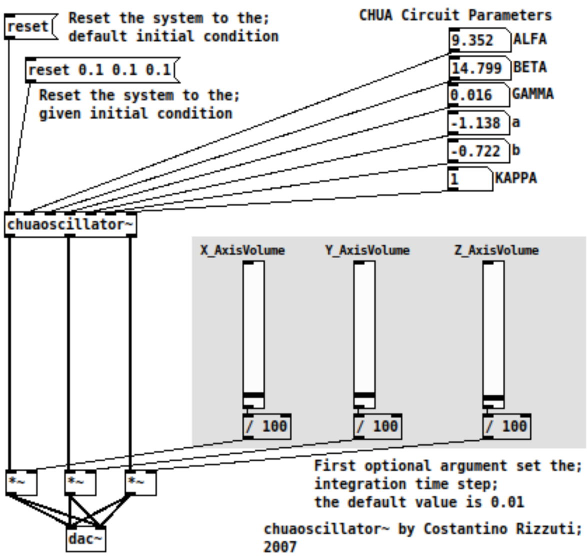

Figure 1: *The help patch of the audio rate Chua's oscillator.*

Most signal processing environments for computer music, as well Pd, transfer audio data among their objects by using vectors (in Pd they are called *blocks*). As stated by Yadegari [12], in an environment which transfers audio among its objects in blocks, to solve a system of differential equations either we have to write an external object which performs the calculation for every sample or create a patch setting the block-size equal to 1. Yadegari proposes a third solution to this problem through the *fexpr∼* object of Pure Data and Max/MSP that allows to calculate numerical expressions for each audio sample. The method proposed by Yadegari for the numerical integration of differential equations within visual development environments such as Pd and Max/MSP is very interesting and useful especially for its ease of use. The limit of this approach consists in the use of the implicit Euler method for the numerical integration of differential equations. This method certainly offers the advantage of simplicity of calculation, but has two important defects: it is not very accurate, for the same integration step, compared to other methods such as for example the Runge-Kutta method; it can often be unstable when the function that characterizes the differential equation is characterized by a high degree of variability. Both of these characteristics make the Euler method not so suitable for the numerical integration of systems that exhibit chaotic behavior. In fact, the low accuracy of the numerical integration method together with the well-known property of chaotic systems of sensitivity to conditions determines the growth and propagation of calculation errors that can lead to obtaining strongly divergent trajectories even after a short time interval. Two Pure Data externals have been created applying the fourth-order Runge-Kutta method to perform the numerical integration of the dimensionless equations of the Chua's circuit, producing audio signals in one case and control data in the other one.[1]

The Pd externals receive as input the values of the dimensionless parameters mentioned above and returns the values of the coordinated x, y, z in three different outputs. In the case of the control data object the integration process is realized every time the leftmost inlet of the object receive a "*bang*" message. The three coordinate of the new computed point are sent out by the three outputs of the object. For the DSP object the integration process is realized at "audio time" computing a new point in the circuit's space of phases for each audio sample, in this way it's possible to obtain from each external's output an audio signal that can be easily used inside any signal processing network in Pd. Fig. 1 shows the help patch of the audio rate Chua's oscillator external. Sending the "*reset*" message resets the initial conditions at the point specified by the three arguments of the message that define the coordinates x, y, x of the point. Sending a message without arguments the initial conditions are set to a default value ($x = y = z = 0.001$). The following code shows the application of Runge-Kutta for the numerical integration of the differential equations shown in equation 1. The two externals can receive an argument that allows to define the time step of the numerical integration process of the system of differential equations. If no value is provided to the first parameter, the externals use a default value equal to 0.01. In the following code, the variable h is used to store the time step value, while the variables $h2$ and $h6$ are equal to half and one sixth of the time step value.

$$
\begin{aligned}
&//RoundOne\\
&Fx = b * x + .5 * (a - b) * (abs(x + 1) - abs(x - 1));\\
&k1[0] = \kappa * \alpha * (y - x - Fx);\\
&k1[1] = \kappa * (x - y + z);\\
&k1[2] = \kappa * (-\beta * y - \gamma * z);
\end{aligned}
$$

$$
\begin{aligned}
&//RoundTwo\\
&temp0 = x + h2 * k1[0];\\
&temp1 = y + h2 * k1[1];\\
&temp2 = z + h2 * k1[2];\\
&Fx = b * temp0 + .5 * (a - b) *\\
&\qquad (abs(temp0 + 1) - abs(temp0 - 1));\\
&k2[0] = \kappa * \alpha * (temp1 - temp0 - Fx);\\
&k2[1] = \kappa * (temp0 - temp1 + temp2);\\
&k2[2] = \kappa * (-\beta * temp1 - \gamma * temp2);
\end{aligned}
$$

[1] Codes and examples of the two Pd externals can be found here: http://www.costantinorizzuti.com/chuasoscillator

$//RoundThree$
$temp0 = x + h2 * k2[0];$
$temp1 = y + h2 * k2[1];$
$temp2 = z + h2 * k2[2];$
$Fx = b * temp0 + .5 * (a - b) *$
$\quad (abs(temp0 + 1) - abs(temp0 - 1));$
$k3[0] = \kappa * \alpha * (temp1 - temp0 - Fx);$
$k3[1] = \kappa * (temp0 - temp1 + temp2);$
$k3[2] = \kappa * (-\beta * temp1 - \gamma * temp2);$
$//RoundFour$
$temp0 = x + h * k3[0];$
$temp1 = y + h * k3[1];$
$temp2 = z + h * k3[2];$
$Fx = b * temp0 + .5 * (a - b) *$
$\quad (abs(temp0 + 1) - abs(temp0 - 1));$
$k4[0] = \kappa * \alpha * (temp1 - temp0 - Fx);$
$k4[1] = \kappa * (temp0 - temp1 + temp2);$
$k4[2] = \kappa * (-\beta * temp1 - \gamma * temp2);$

$x = x + (k1[0] + 2 * k2[0] + 2 * k3[0] + k4[0]) * h6;$
$y = y + (k1[1] + 2 * k2[1] + 2 * k3[1] + k4[1]) * h6;$
$z = z + (k1[2] + 2 * k2[2] + 2 * k3[2] + k4[2]) * h6;$

$*out3 + + = z;$
$*out2 + + = y;$
$*out1 + + = x;$

Figure 2 shows the waveforms produced by the double scroll attractor called DE1 by Bilotta and Pantano [13]. In this work the authors present 150 chaotic attractors produced by the numerical simulation of the dimensionless shape of the Chua's oscillator derived from 19 sets of parameters that define as many attractors defined as basic shapes.

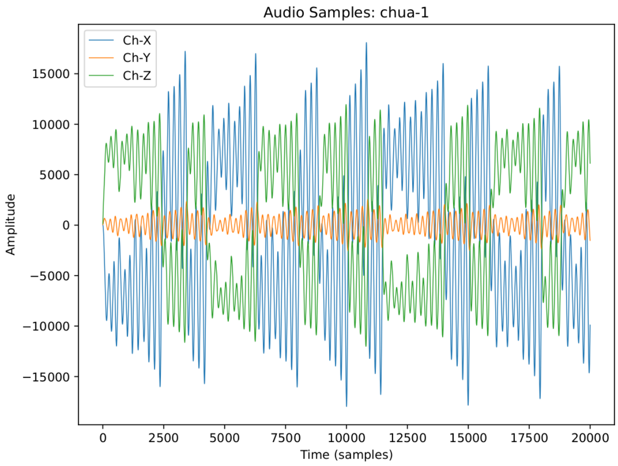

Figure 2: *The waveforms produced by the double scroll attractor.*

The set of parameters for the double scroll attractor are shown in the first row of Table 1. Figure 3 shows the waveforms produced by the single scroll attractor obtained using the set of parameters in the second row of Table 1. Figure 4 shows the waveforms produced by the set of parameters in the last row of Table 1. The behavior of the dynamical system in this case is a fixed point that is reached starting from the initial conditions after a certain interval of time. The image in figure 4 shows the typical time evolution of a damped periodic oscillator.

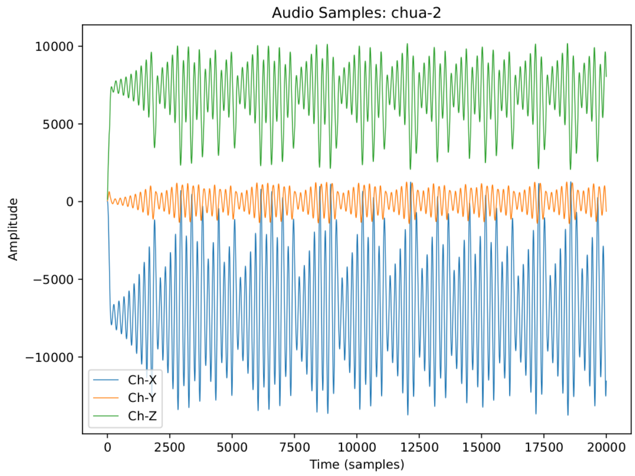

Figure 3: *The waveforms produced by the single scroll attractor.*

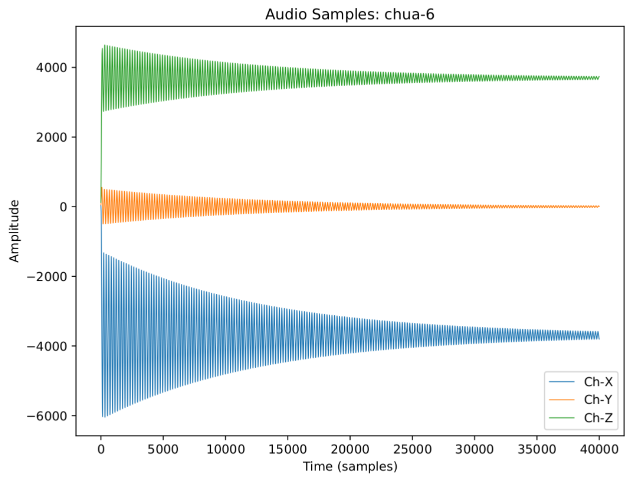

Figure 4: *The waveforms produced by the convergence to a fixed point.*

The control rate external outputs a bang on the fifth output in the case the oscillator trajectory in the phase space becomes divergent and the numerical integration process is interrupted. In this case it is necessary to send a "*reset*" message again to reset the initial condition of the system and probably check the value of the system control parameters because they could be such as to define a condition of unstable behavior of the dynamical system. The external that realize the simulation of the Chua's oscillator at audio rate does not have a dedicated output to signal the divergence of the trajectory. If this circumstance occurs, the process of calculating the new points of the trajectory is interrupted and the external prints the text "*OVERFLOW*" on the Pd console. The verification of the divergence of the trajectory is carried out in the code by checking that the value of the three state variables defining the state of the system does not exceed the value of 100.

Table 1: *The value of the control parameters for the attractor DE1.*

α	β	γ	a	b	k
9.35159	14.79032	0.01607	−1.13841	−0.72245	1
8.5	14.79032	0.01607	−1.13841	−0.72245	1
7.7	14.790312	0.01607	−1.13841	−0.72245	1
7.0815	14.790312	0.01607	−1.13841	−0.72245	1
7.081	14.790312	0.01607	−1.13841	−0.72245	1
7.0	14.790312	0.01607	−1.13841	−0.72245	1

3. LISTENING TO CHAOS

Many approaches have been used to transform in music and sounds time series produced by dynamical systems. Mainly these approaches can be divided in two categories: high level, for melodic pattern generation, low level, in which dynamical systems are used to generate sound samples. In this work we focus on this second type of applications in which the time series produced by the numerical integration of the chaotic system are transformed into audio signals through a one to one rendering ratio. By this way we create a direct mapping from numerical data to sound samples, that is the value of one of the coordinates of the new point that identifies the current state of the dynamical system in its phase space is associated with the current sound sample [14, 15, 16].

In particular, the *chuaoscillator∼*, the external that works at audio frequency, produces three audio signals, related to the state variables of the system, that in the vast majority of cases are not compatible with the audio signal standard adopted by Pd. Therefore, it is always necessary to insert an attenuation of the signal amplitude, as shown in Figure 1, to avoid the saturation of the audio signal output by the computer's sound card. Furthermore, the signals produced by the simulator are often characterized by high DC offset values (see Fig. 3 and 4) that can produce problems or unexpected results if these signals are used in well-known synthesis processes. For this reason, it can be useful to use high-pass filters (the *hip∼* object in Pd) to filter the DC component of the signal and possibly also the frequency region below 20-30 Hz, often very rich in energy in the vast majority of chaotic attractors.

Since the Chua's oscillator is characterized by three state variables, its evolution in a three-dimensional space produces three time series related to the axes x, y, z. This raises the problem of how to transform the data produced by the simulator into sound informations. Different solutions are possible, the result of which must be evaluated above all in light of the sound results that one wants to obtain. In fact, it is possible to have three distinct sound signals to be used separately, it is possible to combine pairs of signals (realizing two-dimensional projections of the system trajectory) or to combine the three signals appropriately to obtain a single signal (for example, using the expression of the Euclidean distance to calculate the modulus of the position vector of the point p representing the state of the system $p = \sqrt{x^2 + y^2 + z^2}$). The same considerations can be made when choosing the method to apply the attenuation factor for the amplitude of the signals produced by the simulator. In fact, one can calculate the bounding box of the attractor and apply an appropriate uniform gain reduction on the three axes to obtain a signal in the audio range of audio signals that does not alter the shape of the three-dimensional trajectory. Or one can realize non-uniform normalizations that consider individually the three signals that will produce a deformation of the system trajectory considered as a three-dimensional entity.

3.1. Attractor tuning

The fundamental parameter to choose in the implementation of the numerical integration process is the step size that mainly affects the degree of accuracy of the numerical approximation of the trajectory of the dynamical system in its space of phases. As already observed in several works [14], it is possible to perform a "tuning" process of the chaotic attractors by modifying the integration step size of the simulator. In fact, the frequency varies proportionally to the step size because if this value increases, fewer points are calculated for a fixed sample rate and for a given set of control parameters defining a specific behaviour of the system (limit cycle or strange attractor). Therefore reducing the number of points per cycle the frequency of the signal increases. If the step size value decreases, the frequency of the signal produced decreases as well because more points are calculated for the same trajectory of the dynamical system. Since the independent variable with respect to which the numerical integration process is carried out is time, this quantity is often called *time step*. In the case of the present work, the numerical integration of the dimensionless model of the Chua's oscillator is actually carried out. Therefore the independent variable is not the time t expressed in seconds, but the dimensionless parameter τ defined according to the relation 3:

$$\tau = \frac{t}{|RC_2|} \tag{3}$$

As pointed out by Chua et al. [17] a configuration of parameters of the dimensionless system can correspond to a large number of configurations of electronic circuits characterized by even very different values of the discrete electronic components that make up the electronic circuit. Therefore, in this case it is possible to imagine a relative time, expressed by the variable τ, that characterizes the dimensionless system and that can be transformed into our "absolute" time measured in seconds through the values of the electronic components in the circuit that is being simulated.

3.2. Exploring the parameter space of the Chua's oscillator

As underlined by several authors, the great advantage of using chaotic systems for the production of sounds and music consists in the possibility of exploiting the wide range of temporal behaviors produced by this family of dynamical systems. Chua's oscillator shows a wide range of chaotic phenomena, among which the so-called routes to chaos are of particular interest: a modification of the control parameters that gradually lead the system to change its dynamic state from the manifestation of a fixed point (a stable stationary behavior), through periodic behaviors of increasing complexity until the onset of chaotic phenomena characterized by the emergence of strange attractors with very different shapes.

All these different dynamic behaviors produce a vast collection of sound phenomena that move in equilibrium between the thin border between periodic sounds at a fixed pitch produced by the evolution of limit cycles, complex sounds generated by periodic behaviors with a high period value and more or less colored noise forms, characterized by very different energy distributions, typical of chaotic attractors. Even the phenomenon of convergence towards fixed points can have an interesting use for the production of sounds since it produces the typical behavior of damped oscillators characterized by periodic sounds with amplitude damped in time (see Fig. 4).

This is a very interesting and complex path to explore as underlined by Essl [18] that proposes a graphical methods to explore the parameter space of chaotic systems in order to find configurations that produce interesting sounds. In fact, often the limit in this kind of experimentation consists in the use of standard parameter configurations that are not always able to fully describe the great richness of behaviors provided by a chaotic system.

On the other hand, as pointed out by some authors [9], the problem of musical control of such systems makes them practically unusable as musical instruments based on physical models sound synthesis. In this work a different approach is proposed that aims to use the Chua's oscillator as a generator of complex sound signals to be inserted into sound synthesis and processing systems. A current research path that is being conducted consists in exploring the possibility of creating sounds that have different characteristics over time through the modification of the control parameters that can produce transitions in the dynamic behavior of the system. As a starting point one can try to follow the parameter changes that produce some of the well-known routes to chaos well defined in the literature on the Chua's circuit. The table 1 presents the parameters of the DE1 attractor called double scroll the first row shows the set of parameters that allows to simulate the trajectory of the chaotic attractor. By reducing the value of the variable α, as shown in the following rows, different behaviors are obtained with a reduction of signal complexity. The second and third rows show two ranges of the parameter α where chaotic behaviour still happens. The fourth rows shows a set of parameters corresponding to an high period limit cycles that shows an acoustic effect that seems like an amplitude modulated periodic sound using sawtooth waveform. The fifth rows show a set of parameters that create a simple limit cycle with a stable periodic waveform. All the values of the parameter α between the last two values in the table create damped periodic oscillations with different damping times. In the case of the value shown in the last row the decay time of the periodic sound is very short and the damping effect is fast. When the value of α increases the decay time increases as well until the transition between fixed point and limit cycle occurs and the periodic behavior becomes stable.

4. COMPOSITIONAL APPLICATIONS

4.1. Chaotic Modulator

The Chaotic Modulator is a synthesizer prototype, made in Pure Data, based on the use of the Chua's oscillator in a process of synthesis by frequency modulation (FM). The instrument is polyphonic and implements a classic FM synthesis process in which the time series produced by the numerical simulation of the Chua's oscillator are used as modulating signals of six carrier oscillators.

The instrument is equipped with eight voices each with an amplitude envelope generator, three low pass filters with an envelope generator controlling the behavior of the filters over time and a third envelope generator changing the FM index of modulation along the sound duration. The synth is controlled via the MIDI protocol; when it receives *noteOn/noteOff* type messages it sends the start and stop messages to the envelopes inside the instrument and at the same time determines the value of the fundamental frequency of the sound to be played in order to change the frequency of the oscillators. Figure 5 shows a schematic representation of the synthesis system used to create the Chaotic Modulator.

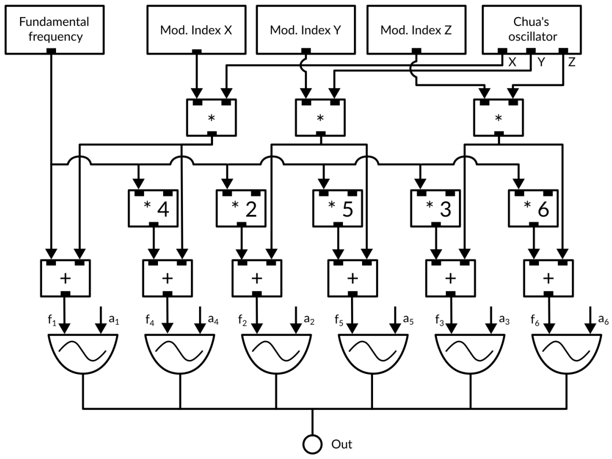

Figure 5: *Schematic of the synthesis system used in the Chaotic Modulator.*

The six carrier oscillators are appropriately tuned to create a harmonic structure with six partials calculated in relation to the fundamental frequency value of the MIDI note received through the reception of the *noteOn* message. The three signals produced by the simulation of the Chua's oscillator relating to the three state variables x, y, z are used to modulate the frequency of the oscillators. In particular, it was chosen to experiment with the following configuration: the signal relating to the x axis modulates the first and fourth partial, the one relating to the y axis modulates the second and the fifth and finally the one relating to the z axis modulates the third and the sixth. This association is obviously arbitrary and various other configurations can be adopted considering that in this context the quality of a choice can only be evaluated through the sound quality of the sounds produced by the instrument.

The frequencies of the carrier oscillators have been organized by using integer ratios in order to obtain an harmonic sound in the absence of modulation or for low values of the three modulation indices. This choice is also arbitrary and subject to changes and revisions since it strongly determines the sound behavior of the instrument which in this case, in the absence of modulation, produces sounds that strongly evoke the sounds of electric organs. The fundamental frequency of the synthesizer can be controlled through a keyboard or any other MIDI device.

By controlling the three modulation indices (in the top of figure 5) it is possible to modify the spectral content of the sound, going from a sound with almost exclusively harmonic partials, in the case of low values of the three indices, to a sound with an articulated timbre and with a strong presence of inharmonic and noise components, obtained for high values of the three indices. Figure 6 shows the graphic interface of the synthesizer where it is possible to observe the different parameters that the user can modify to change the nature of the sound produced by the instrument.

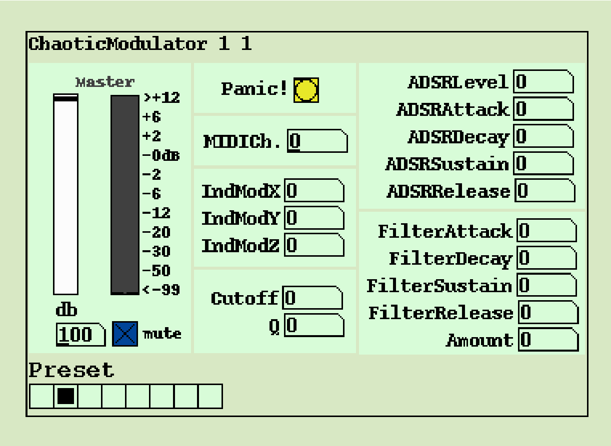

Figure 6: *The graphic interface of the Chaotic Modulator.*

4.2. Noisy silence

Noisy Silence[2] is a live electronics performance, first performed in 2014, that explores the possibilities of using the sounds produced by a chaotic system as basic material for the construction of a musical discourse aimed at investigating the dialectics between silence and sound and between sound and noise. The noisy and tumultuous sounds produced by the chaotic dynamics generated by the numerical simulation of the mathematical model of the Chua's oscillator have been used as basic material to be manipulated through common systems of digital sound synthesis and processing. The sounds produced by the Chua's oscillator can be classified as noises from a perceptive point of view, but, for the richness and variety of sound behaviors that characterize them, they can be very fascinating starting materials on which to operate manipulation and elaboration processes aimed at the search for unheard sounds. During the performance, the sounds derived from the simulation of the chaotic system are accompanied and contrasted by materials that tell, through the reading of texts, the condition of abandonment and depopulation of numerous Calabrian inhabited centers. The "noisy silence" referred to in the title is an allusion to the condition of such places where the silence linked to abandonment and the absence of human presence is transformed in the ear of the attentive visitor into an incessant noise that tells of the continuous work of Nature that transforms and manipulates the territory in a way that is heedless of human action. The performance has an unfixed duration that can vary from a minimum of 10 minutes to a maximum duration ad libitum.

Chaotic signals are mainly noisy, but the characteristics of their noises are distinct from white noise, which has a uniform broad band spectrum. The spectrum of chaotic signals resembles a 1/f energy distribution. In chaotic signals there seem to be pitch characteristics attributable to a centered energy concentration around specific frequency regions, independent of the amount of noise in the signal. For these reasons these signals can be used in an interesting way in various sound synthesis processes.

Three different instruments were developed to set up the performance:

- ChaoticResSynth
- ChaoticSubSynth
- ChoaticRMSynth

The three instruments, whose operation will be described in detail below, send their synthesized sounds both to the *dac∼* object of Pd to send out the sound to a sound card and to another patch providing a very long delay line with a time whose value varies over time throughout the entire performance from a maximum value of 48 to a minimum of 3 seconds. The use of the delay line is aimed at creating sound structures that periodically return over time and that, appropriately controlled especially through the modification of the feedback parameter, create mechanisms of accumulation and thickening of the sound materials for high feedback values and thinning when the feedback value decreases.

All the instruments are provided with an external control system based on Control Change MIDI messages, allowing to simultaneously control the different parameters in real time while performing.

4.2.1. ChaoticResSynth

In this instrument the three signals produced by the numerical simulation of the Chua's oscillator are used as excitation signals of three resonators built using Feedback Delay Network (FDN) [19, 20]. The three resonators consist of eight delay lines tuned to resonate on harmonics of three different pitches. The first resonator is tuned to the note C2 with a fundamental frequency of 65.4 Hz, the second one to the note C4 with a frequency of 261.63 Hz, and the third to the note G3 with a frequency of 195.99 Hz. The characteristics of the sound produced by the resonators depend mainly on two parameters: the feedback of the delay lines and the rotation angle that controls the behavior of the matrix that carries out the signal circulation between the eight delay lines. The effect of the values chosen for the feedback is well known and allows to obtain sounds with very long decays in the case of values close to 1 and increasingly shorter decays for lower feedback values. The signal distribution matrix between the different delay lines uses the mathematical concept of rotation expressed in practice through the the use of cosine and sine functions. A rotation angle equal to 0 makes the delay lines independent whose signal recirculates only on themselves, on the contrary a value of 90 degrees connects the delay lines in series by circulating an initial signal through all the delay lines before returning in feedback to the beginning. The sonic behavior determined by the change of the rotation parameter is much more complex to explore, but often provides very interesting and sometimes surprising results.

The three signals produced by the oscillator are all processed according to the same processing chain described below:

- Ring Modulation (RM)
- Amplitude envelope
- FDN Resonator
- Gain control
- Stereo panning.

In this sound synthesis scheme the signal produced by the chaotic system is used as an exciting signal for the resonators based on the Feedback Delay Network. The use of Ring Modulation (RM) is very useful to realize a shift on the frequency axis of the sound produced by the Chua's oscillator which is generally

[2] More information and audio recordings of the performance are available at the following link: `http://www.costantinorizzuti.com/noisysilence`

characterized by low frequency oscillations. The application of RM allows to realize a translation of the chaotic spectrum on the frequency axis (especially towards high frequencies) which makes the interaction with the resonator based on the FDN technique much more interesting. The RM technique, also called Double Side Band Modulation (DSB), also produces an evident enrichment of the frequency spectrum due to the emergence of a spectral band symmetrically flipped downwards with respect to the carrier frequency which determines a better response of the resonator which receives a signal characterized by a highly complex and time-changing frequency spectrum.

The application of the amplitude envelope is useful to produce a sequence of sound events controlled by a Pd *metro* object characterized by a precise temporal duration and by an impulsive amplitude envelope useful to excite the resonator. The sound result produced by this synthesis system is similar to the sequence of percussive sounds produced by strings or metal plates. The function of the chaotic system for the generation of the excitation signal of the FDN is essential to generate sequences of impulses that change over time especially with regard to the frequency content and the amplitude dynamic. Also the control of the frequency of the oscillator that implements the RM is a very useful factor for the control of the sound results. The choice of the frequency of the carrier oscillator of the RM controls the frequency content of the sound produced by the resonator especially with regard to the initial instants of attack of the sound.

4.2.2. ChaoticSubSynth

The instrument implements a subtractive synthesis system based on a bank of twenty eighth-order band-pass filters. The filters can operate in two different modes: continuous and periodic. In the first case, a normal signal filtering is applied, in the second case, a periodic amplitude envelope is applied to the filtered signal based on the reading of a table containing a Gaussian bell-shaped curve. The behavior of the periodic operating mode can be modified through two main parameters: the reading period of the table and the overall duration of the sound event. In this way, the instrument is able to generate materials similar to what is obtained with granular synthesis systems. The instrument has a certain number of different tables containing Gaussian curves of different shapes: from those with a very wide bell to those on the contrary with a very narrow bell. The input signal sent to the filter bank is obtained by adding the outputs of many sources that can be mixed together to create a very complex wave: the three signals generated by the Chua's oscillator, a white noise generator and a sawtooth oscillator. The signals produced by the numerical simulation of Chua's oscillator are all subjected to a Ring Modulation process before to be added with the other signals. This transformation is useful for transposing their frequency spectrum into different frequency regions that can be defined through the frequency of the carrier oscillators. The output of the filter bank is sent to the Pd *rev3∼* object to add some artificial reverberation and then the original and reverberated signals are summed and sent to the output.

The choice of the type of filter behavior make it possible to create two different sound behaviors that are very important in the construction of the artificial soundscape proposed in the performance. In fact, in continuous mode the filter produces static textures that are useful for evoking the sound of environmental elements such as the blowing of the wind and the flowing of far water. Different presets have been created that define the tuning of the filters and the Q values that define the width of the filter's pass band. The filters have a system of interpolation over time of these two parameters, so changing the preset produces interesting glissando effects of the filter frequencies and widening or narrowing of the frequency bands that characterize the signal produced by the instrument. In periodic mode, a sound granulation process is produced that is very useful for evoking the sounds of insects and other natural agents that produce sounds of a granular nature or composed of a large number of small sound events. In the granulation process, the duration of the sound events is chosen randomly according to a Gaussian distribution based on the parameter set in the preset that defines the average granulation period. A second parameter controls the dispersion of values around the mean value.

4.2.3. ChoaticRMSynth

This instrument was developed to insert audio files into the performance and in particular the recording of the reading of texts that tell the condition of abandonment and depopulation of numerous Calabrian small towns. The reciting voice was imagined as a floating element that at times must have a narrative and descriptive power through the intelligibility of the words of the text and, at other times, must be a purely sonic element that, while always evoking speech, manages to accompany, intertwine and contrast with the materials produced through the synthesis systems presented previously. The instrument performs the ring modulation between the sounds produced by the numerical simulation of the Chua's oscillator and the reading of sound files stored in Pd tables. Three identical sub patches have been created performing the same sound elaboration, one for each of the three state variables x, y, z of the chaotic oscillator. Each of these sub patches is configured with its own configuration parameter to create sound variety while using the same sound processing process. The parameter values have been selected and saved to create a set of presets. The choice of values for the different parameters of the sub patches was made mainly by evaluating the sound characteristics of the signals produced by the three coordinates of the dynamic system and the type of sound result that was desired for a given preset.

The three signals produced by the chaotic oscillator are all processed according to the same processing chain described below:

- Frequency shift (SSB Mod)
- Band pass filtering
- Ring Modulation with the sound file signal
- Low pass filtering
- Dry/wet balancing
- Gain control
- Stereo panning.

During the development phase of this instrument, the use of RM was experimented to translate the spectral content of the signal produced by the simulation of the Chua's oscillator along the frequency axis. In this case, it was considered that this operation did not provide good results because the signal obtained in this way was excessively complex and with a rich energy spectrum that was not suitable to be used as a signal to be subjected to further ring modulation with audio files containing speech. For this reason, it was decided to use a frequency shift operation using a technique called Single Side Band (SSB) modulation [20].

Furthermore, the signal after the dry/wet balancing stage is split into two parallel paths, one going to the next gain control stage and the other to be used as an excitation signal for a resonator like the ones previously presented for the *ChaoticResSynth* patch. For all the three sub patches the resonator is tuned to the note C2 with a fundamental frequency of 65.4 Hz. The two signals, the original and the resonator output, are finally added to the input of the gain control stage.

5. CONCLUSIONS

A review of compositional application developed in the last years using a chaotic dynamical system in different sound synthesis processes have been presented. Two externals object for Pd were developed to realize the numerical simulation of the Chua's oscillator equations by using the fourth order Runge-Kutta method. The three audio signals produced by this object was used within different sound synthesis methods leading to the creation of a family of new musical instruments that exploit the potential offered by the use of a chaotic system to produce complex sounds. All these instruments have been used for the creation of musical pieces and for the realization of live electronics performances.

In future works I hope to present some new results on an interesting hypothesis about the possibility of transposing the sound produced by the attractors onto different musical pitches. In fact, if we can tune the oscillator so that it provides a specific pitch (for example a C note) then with a pitch shifter we can transpose the sound onto all the other pitches and therefore play the system using the tempered system or any other intonation system. Moreover I hope to obtain also interesting results on the generation of sound with complex time evolution by working on the dynamic changes of the Chua's oscillator parameters.

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